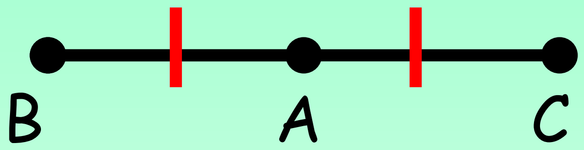
Medians, Altitudes, and Perpendicular Bisectors

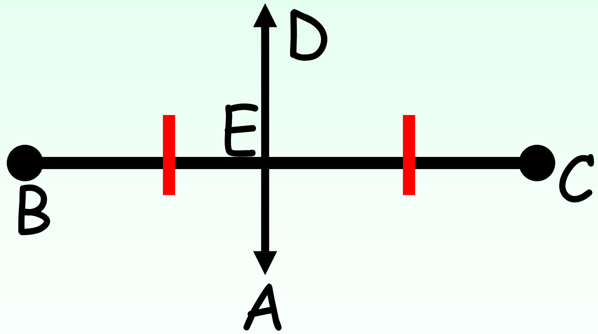
Recall the following definitions:

<u>Midpoint</u> – the point on a segment that divides the segment into two congruent segments.



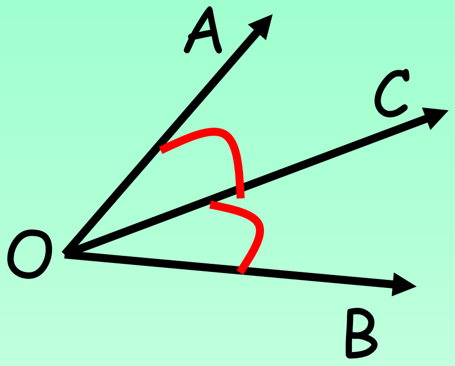
If A is the midpoint of $\overline{BC}$, then $\overline{BA} \cong \overline{AC}$.

<u>Segment Bisector</u> – a line (or segment, or ray) that intersects a given segment at its midpoint.



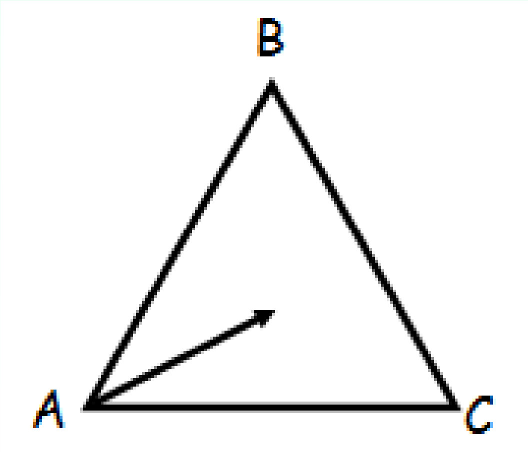
If $\overleftrightarrow{DA}$ bisects $\overline{BC}$, then E is the midpoint of $\overline{BC}$. ($\overline{BE} \cong \overline{EC}$).

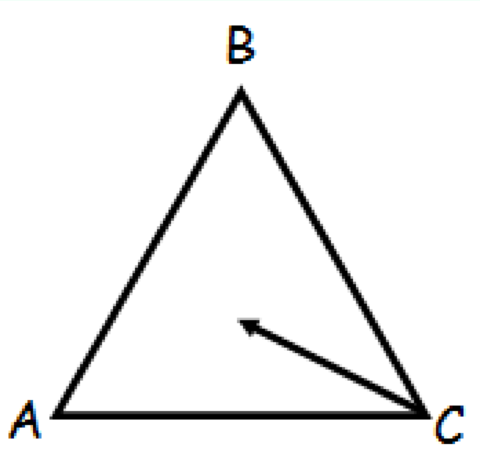
Angle Bisector – a ray that divides the angle into two **congruent**, adjacent angles.

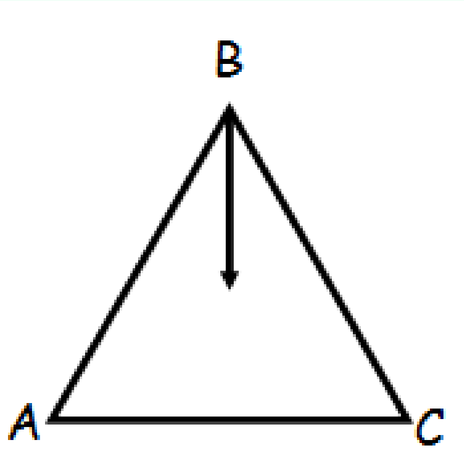


If $\overrightarrow{OC}$ bisects $\angle AOB$, then $\angle COB \cong \angle AOC$.

Every triangle has **three** $\angle$ bisectors because there are three angles.

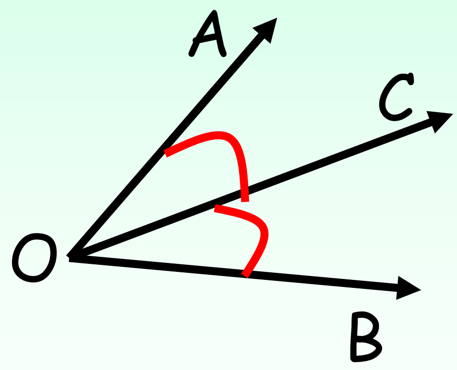




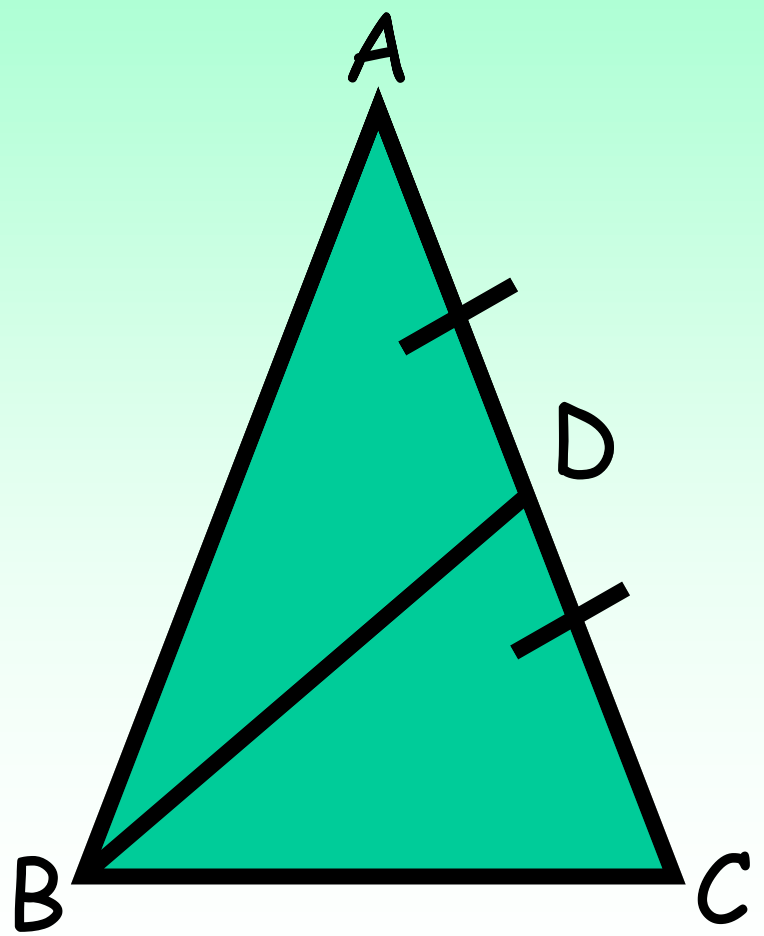


To identify an angle bisector,

<u>look for two congruent angles.</u>

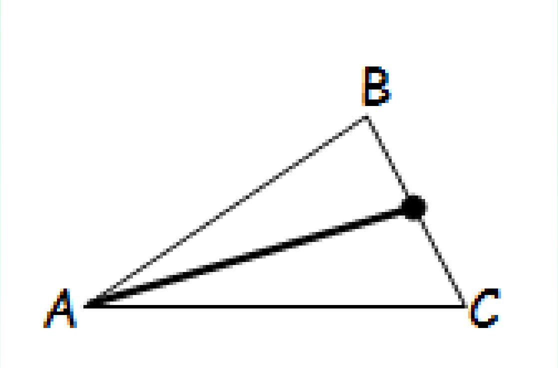


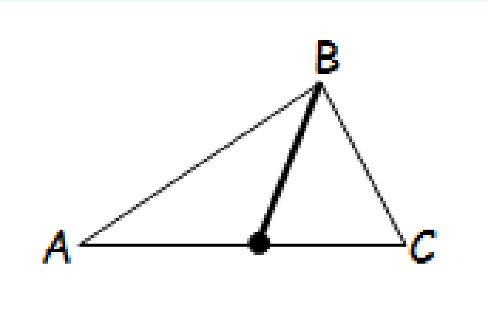
Median: In a triangle, a segment from a **vertex** of the triangle to the **midpoint** of the opposite side.

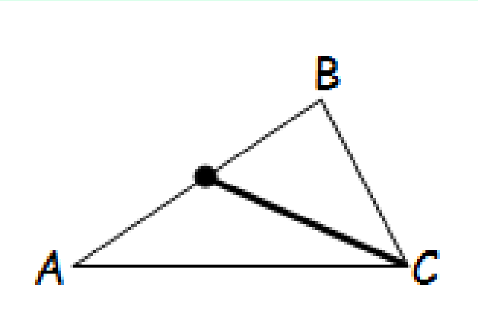


If $\overline{BD}$ is a median of ΔABC, then D is the midpoint of $\overline{AC}$.

Every triangle has **three** medians because there are three vertices and three sides.



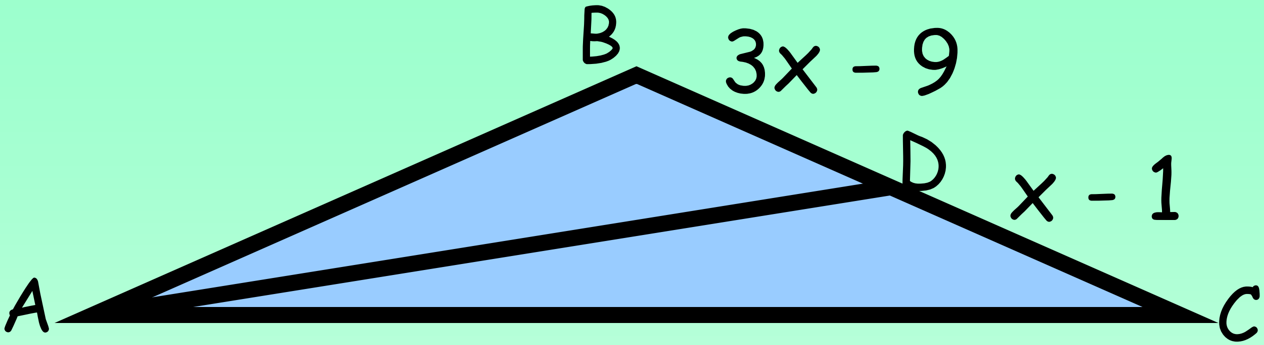




To identify a median,

<u>look for two congruent segments.</u>

Example 1 – Algebra Connection



If $\overline{AD}$ is a median of $\triangle ABC$. Find x, BD, DC, and BC.

Because $\overline{AD}$ is the median, BD = DC.

$3x - 9 = x - 1$ $\quad$ $2x = 8$ $\quad$ $x = 4$

$BD = 3(4) - 9 = 3$

$DC = 4 - 1 = 3$ $\quad$ $BC = 3 + 3 = 6$

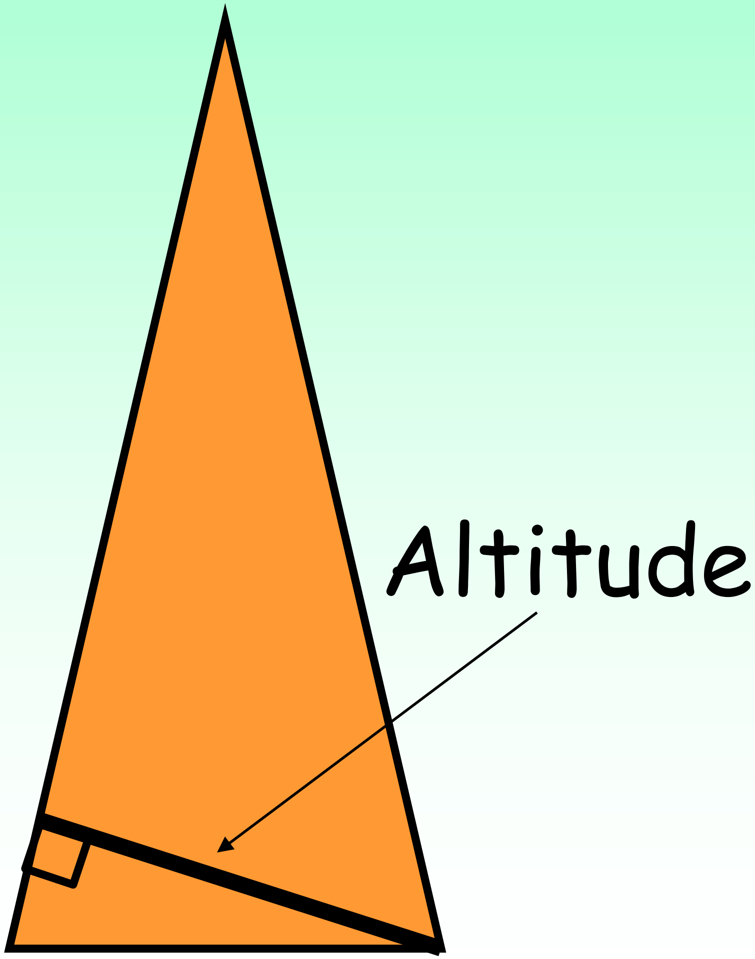
$x = 4$

$BD = 3$

$DC = 3$

$BC = 6$

Altitude: In a triangle, a **perpendicular** segment from a **vertex** to a line that contains the opposite side.

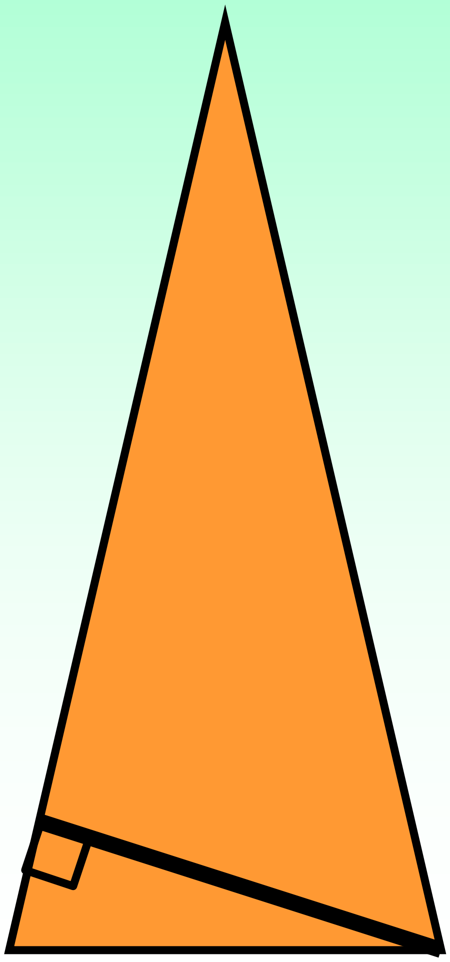


~Every triangle has **three** altitudes because there are three vertices.

~Altitudes can be found inside or outside the △ or can be a side of the △.

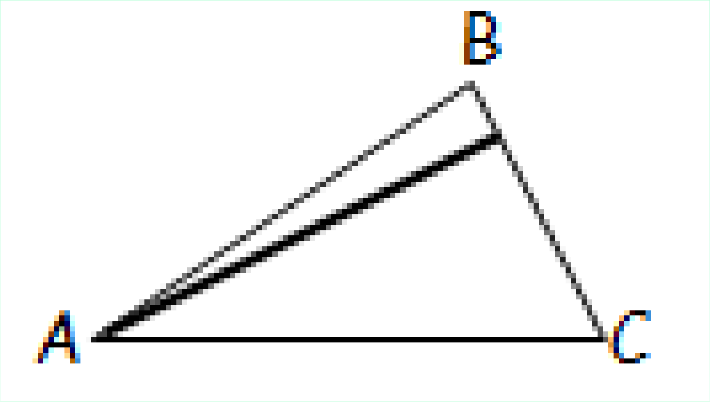
To identify an altitude,

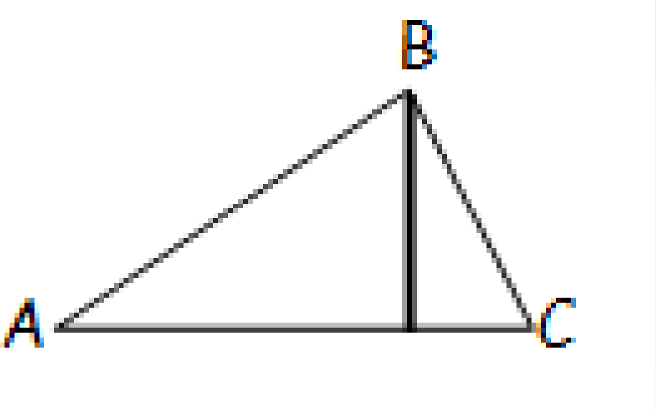
look for a Right Angle.

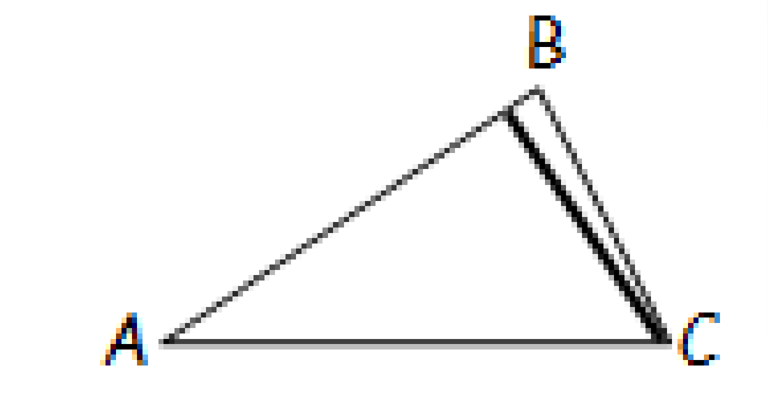


Acute Triangle:

All three altitudes are found <u>inside</u> the triangle.



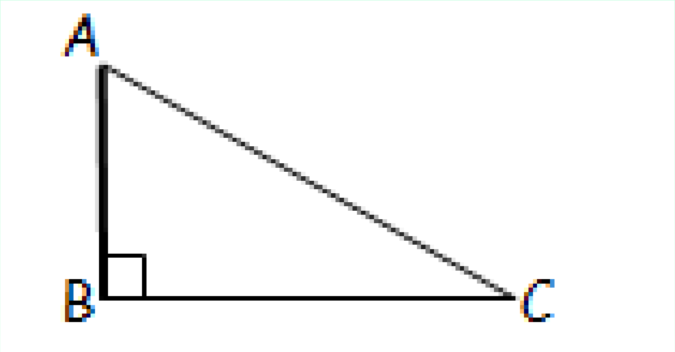


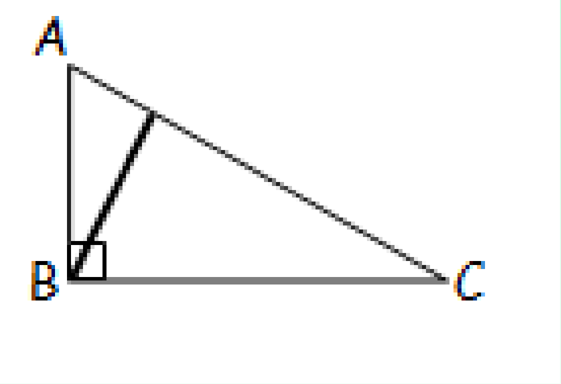


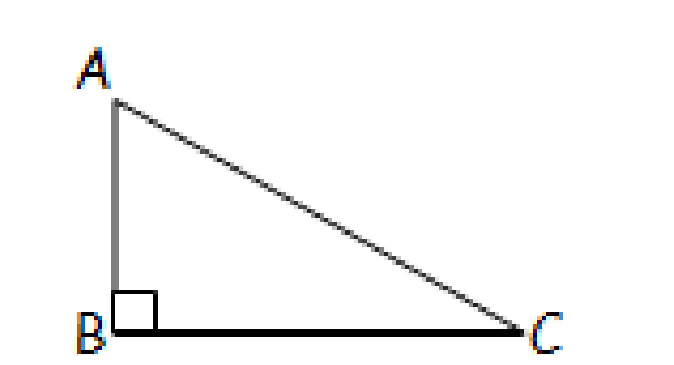
Right Triangle

~Two of the altitudes are <u>part</u> of the triangle (the **legs** of the triangle).

~The third is <u>inside</u> the triangle.



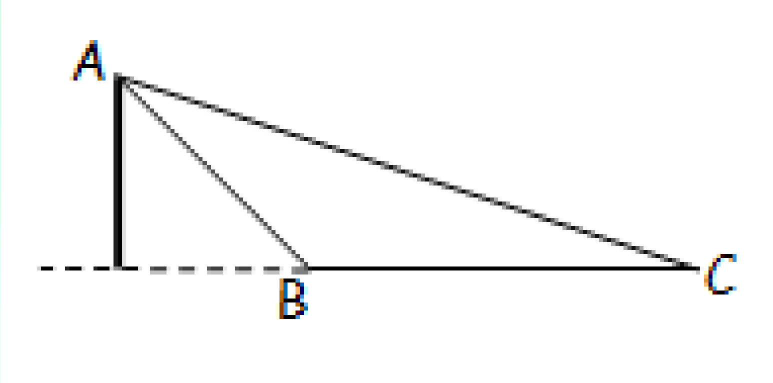


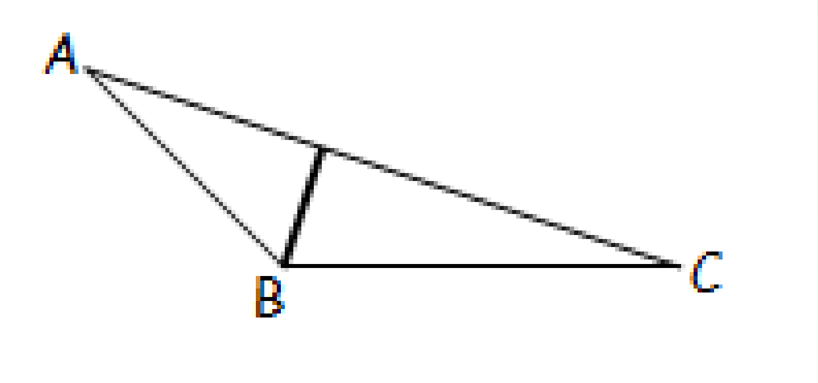


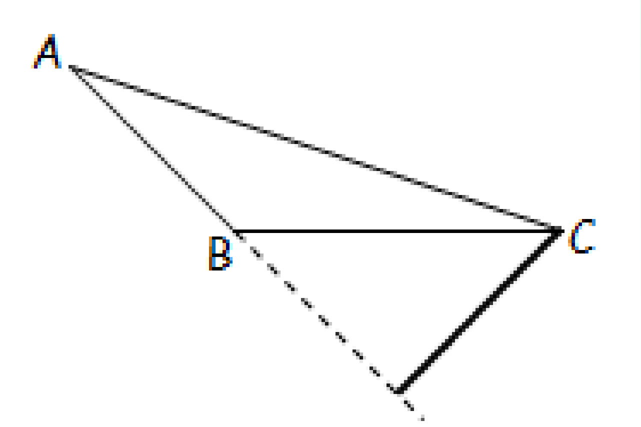
Obtuse Triangle

~Two of the altitudes are outside the triangle.

~The third is inside the triangle.





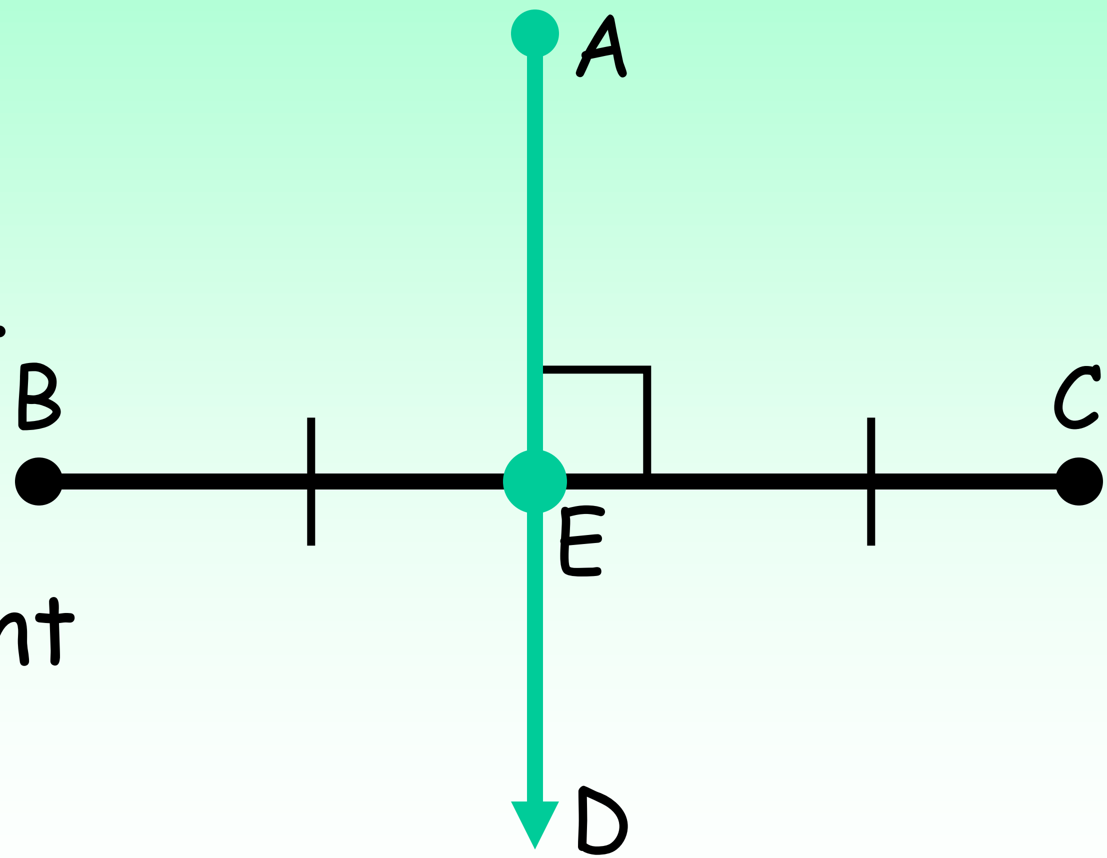


Perpendicular Bisector:

A line (or a ray or a segment) that is **perpendicular** to a segment at its **midpoint**.

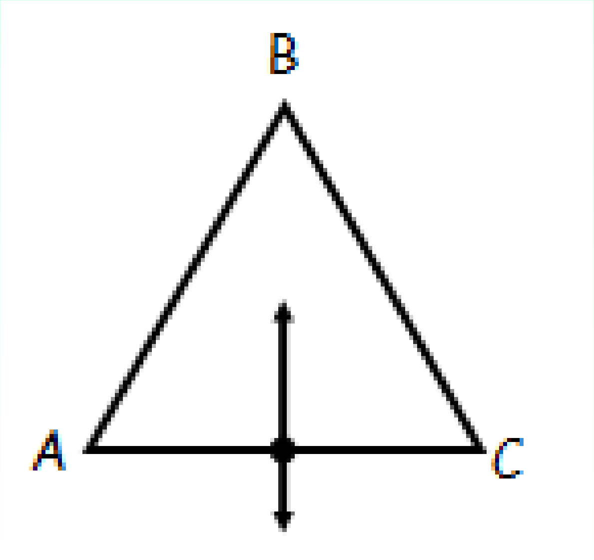
$\overrightarrow{AD}$ is the perpendicular bisector of $\overline{BC}$.

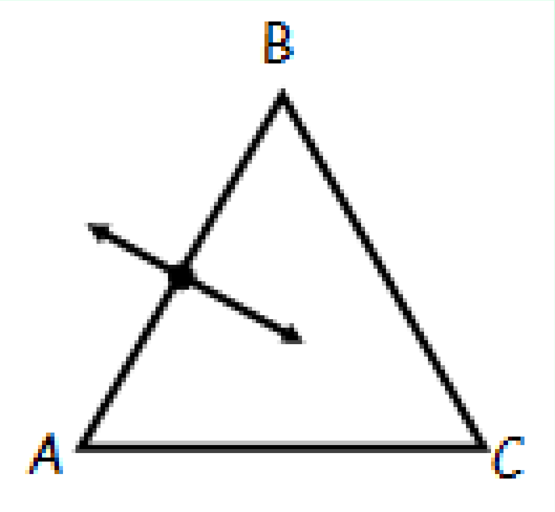
E is the midpoint of $\overline{BC}$.

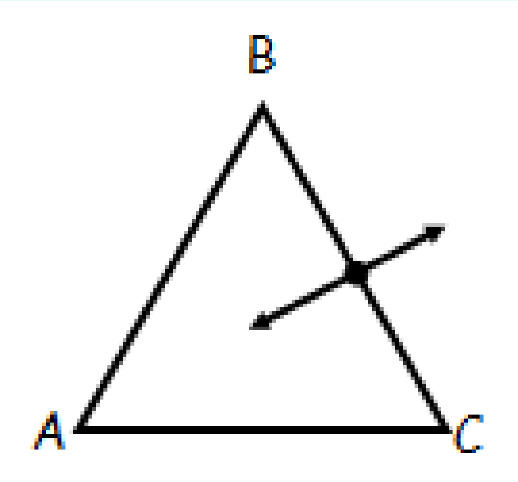


~ Cuts the segment into two equal parts (def. of midpoint)

~ Every triangle has **three** $\perp$ bisectors because there are three sides.

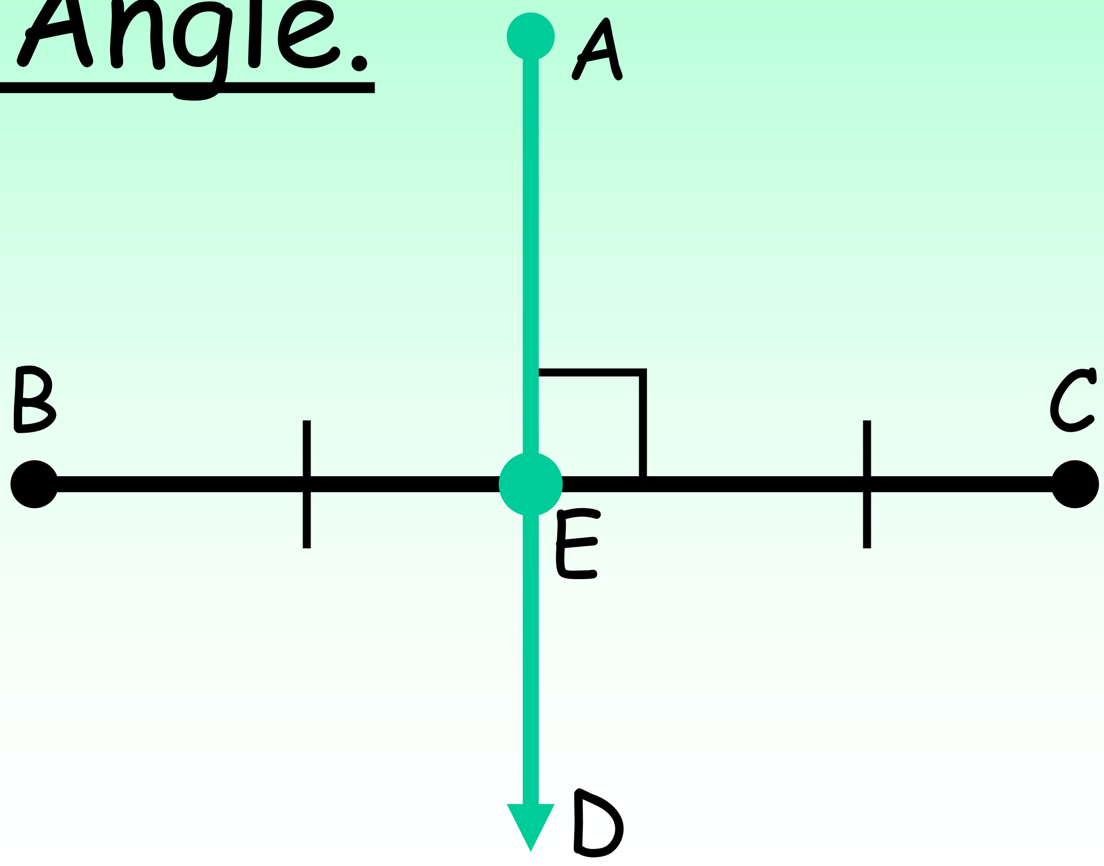






To identify a $\perp$ bisector,

<u>look for Congruent Segments and a Right Angle.</u>



Chart

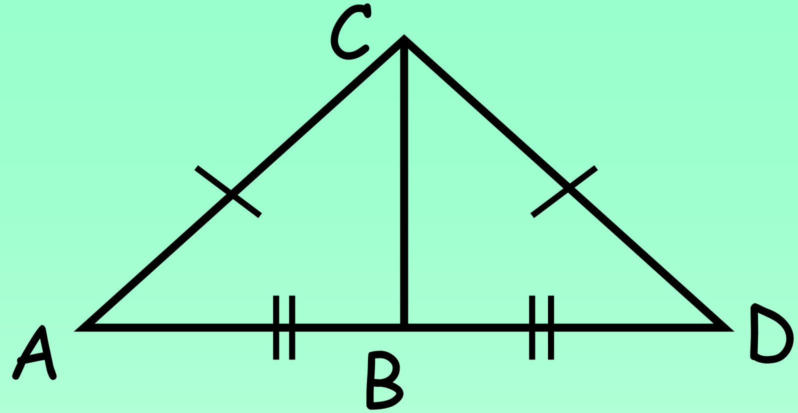
	Goes Through **Vertex**	Forms a **Right Angle**	Goes Through **Midpoint**	Splits angle in **two congruent angles**
Angle Bisector				
Median				
Altitude				
Perpendicular Bisector				

Connections to Proofs…

Given: $\overline{CB}$ is a median of ΔACD

$\overline{AC} \cong \overline{CD}$

Prove: $\overrightarrow{CB}$ bisects $\angle ACD$



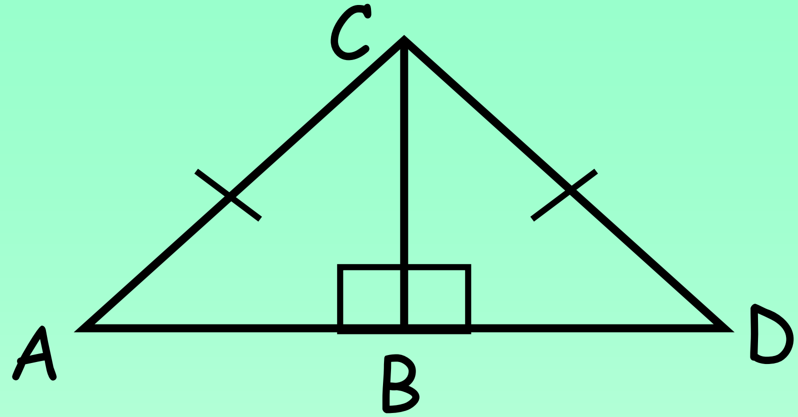
1. $\overline{CB}$ is a median of ΔACD	1. Given
2. B is the midpoint of $\overline{AD}$	2. Definition of a median
3. $\overline{AB} \cong \overline{BD}$	3. Definition of a midpoint
4. $\overline{AC} \cong \overline{CD}$	4. Given
5. $\overline{CB} \cong \overline{CB}$	5. Reflexive
6. $\Delta BAC \cong \Delta BDC$	6. SSS
7. $\angle ACB \cong \angle DCB$	7. CPCTC
8. $\overrightarrow{CB}$ bisects $\angle ACD$	8. Definition of an angle bisector

Connections to Proofs…

Given: $\overline{CB}$ is an altitude of ΔACD

$\overline{AC} \cong \overline{CD}$

Prove: B is the midpoint of $\overline{AD}$



1. $\overline{CB}$ is an altitude of ΔACD	1. Given
2. $\overline{CB} \perp \overline{AD}$	2. Definition of an altitude
3. $\angle ABC$, $\angle DBC$ are right $\angle$s	3. Definition of Perpendicular Lines
4. $\overline{AC} \cong \overline{CD}$	4. Given
5. $\overline{CB} \cong \overline{CB}$	5. Reflexive
6. $\Delta BAC \cong \Delta BDC$	6. HL
7. $\overline{AB} \cong \overline{BD}$	7. CPCTC
8. B is the midpoint of $\overline{AD}$	8. Definition of a midpoint